Spreadsheet for calculating power loads on bending magnet beam line optical components

M. R. Howells 9-8-99

Introduction

The purpose of this report is to explain the calculation of beam line power loads which is incorporated in the Excel spreadsheet prepared by the author. The need to know the power loads is a common requirement in planning beam lines and it will be useful for everyone to have immediate numbers without a need to recreate the calculation for every new beam line project. For bending magnets the emitted heat load is approximately Gaussian-distributed in the vertical and constant over a prescribed width in the horizontal. To carry out a finite element analysis is necessary to know the illuminated width in the horizontal direction and the on-axis strength i. e. the power density and the rms width of the Gaussian in the vertical direction. These quantities are referred to a surface receiving the beam at normal incidence. This is a straightforward calculation of standard synchrotron radiation functions for the first component to receive the beam. However, once the beam has passed a component, which absorbs some of the x-rays, the situation changes somewhat. The height of the Gaussian is reduced in a way that depends on x-ray energy so one has to calculate the absorption and transmission (or reflection) at every energy and sum the results. Since the various x-ray energies have different (Gaussian) spreads, one has also to calculate a new value for the Gaussian width of the total power load that takes into account the removal of different fractions of the radiation at different energies (widths). Thus the first window absorbs most of the lowest-energy x-rays which have the widest spread. This leads to a wide power deposition into that window and a much narrower power distribution emerging from it.

Calculation of the power load on the first component

The calculation proceeds in the following steps. Where possible the equations are referenced to the ALS Handbook (ALSHb) and the notation is the same as ALSHb.

· Calculate the flux per unit solid angle (in ph/sec/mr^2/0.1%BW) on a suitable grid of x-ray energies using ALSHb eqs. 3-15 and 3-16

$$\left.\frac{d^2F}{d\theta\, d\psi}\right|_{\psi=0} [\mathrm{ph / sec / mr^2 / 0.1\%BW}] = 1.327 \times 10^{13} E^2[\mathrm{GeV}] I[\mathrm{A}] H_2(y)$$

where $y = \varepsilon / \varepsilon_c$, $\varepsilon_c[\mathrm{keV}] = 0.665\, E^2[\mathrm{GeV}]B[T]$ and $H_2(y) = y^2 K_{2/3}^2(y/2)$ and K is a special Bessel function.

· Calculate the flux integrated over the vertical angle (that is over ψ) for each x-ray energy using ALSHb eq. 3-17 and 3-18

$$\frac{dF}{d\theta} [\mathrm{ph / sec / mr / 0.1\%BW}] = 2.457 \times 10^{13} E[\mathrm{GeV}] I[\mathrm{A}] G_1(y)$$

where $G_1(y) = y \int_y^{\infty} K_{5/3}(y')dy'$

· The special Bessel functions and their integrals are calculated according to a scheme described by Kostroun [Nucl. Instrum. Meth. **172**, 371 (1980)] in which both are approximated by rapidly converging power series. In the present calculation the process is implemented in an Excel macro the macrosheet for which must run with the spreadsheet.

· The rms width $\sigma_\psi(y)$ at each x-ray energy can now be calculated from ALSHb eq. 3-19

$$\sigma_\psi(y) = \frac{1}{\sqrt{2\pi}} \frac{\frac{dF}{d\theta}}{\left.\frac{d^2F}{d\theta d\psi}\right|_{\psi=0}}$$

· After conversion from numbers of photons to power units (W/mm^2), we can now integrate the power density over all x-ray energies and obtain the required total on-axis power density incident on the first component. In making this summation we take advantage of the calculation of Green [Fig 10] which shows that the energy range $0.01\varepsilon_c < \varepsilon < 7\varepsilon_c$ is sufficient to account for greater than 99% of the power. The rms width of the Gaussian distribution of the total power is then calculated by the following weighted average.

$$\sigma_{ptot} = \frac{\sum_i \sigma_{\psi i} w_i}{\sum_i w_i} \quad \text{where} \quad w_i = \left.\frac{d^2F}{d\theta d\psi}\right|_{\psi=0,i}$$

This completes the characterization of the power load on the first component.

Computational checks for accuracy of the power load on the first component

The following accuracy checks have been applied.

- The calculations in 1. and 2. have been checked against the bending magnet calculations in ALSHb.
- The total on-axis power density obtained by summation over x-ray energies is checked in the spreadsheet against the total power density as given by Kim [Opt. Eng. **34**, 342 (1995)] eq. 24.

$$\left.\frac{d^2P}{d\theta\, d\psi}\right|_{\text{on axis}} [\mathrm{W / mr^2}] = 5.42B[\mathrm{T}]E^4[\mathrm{GeV}]I[\mathrm{A}]$$

- The total power $P_T[\mathrm{kW / mr}]$ obtained by summation over x-ray energies is checked in the spreadsheet against the standard total power formula (ALSHb 3-39 applied to a bending magnet)

$$P_T[\mathrm{kW}] = 1.27E^2[\mathrm{GeV}]B_0^2[T]L[\mathrm{m}]I[\mathrm{A}]$$

where the relation $\rho[\mathrm{m}] = 3.335E[\mathrm{GeV}]/B[\mathrm{T}]$ was used to obtain the trajectory length L per milliradian.

- The calculation of σ_ψ at the critical energy is compared with the ALSHb expression (eq 3-15a)

$\sigma_\psi(\varepsilon_c) = \dfrac{0.65}{\gamma}$ where γ is the electron beam energy in units of the electron rest energy (0.511 MeV)

- The rms width of the total power distribution calculated by the above weighted average over all x-ray energies is checked against a simple and useful formula given by Green [eq. 87]

$$\sigma_{\mathrm{ptot}} = \frac{0.608}{\gamma}$$

In all these cases the agreement was as good as the precision of the input data entitled us to expect along with the fact that we accounted for >99% of the power but not an exact 100%.

Calculation of the transmission of the windows and the reflectivity of the mirror

To calculate the power load on the first window it is necessary to know the percent absorption (or % transmission) of the window at each x-ray energy. This depends on the x-ray absorption characteristics of the window material and on the thickness of the window. To provide the necessary optical data to enable the absorption to be calculated in an automated way in the spreadsheet for all possible materials would involve the storage of a lot of data. Similar arguments apply to calculating the reflecting efficiency of a mirror. Fortunately there is an easier way. It is necessary to go to the Center for X-ray Optics website at www-cxro.lbl.gov and from the home page click on "X-ray interactions with matter". For finding the transmission efficiency of a window, click on "X-ray transmission of a solid". For finding the reflecting efficiency of a mirror, click on "X-ray reflectivity of a thick mirror". Follow the directions to define the material (noting that chemical symbols and formulas are case-sensitive) and request data on the same grid of x-ray energy values adopted above covering at least $0.01\varepsilon_c < \varepsilon < 7\varepsilon_c$. For a window give the thickness. For a mirror give the rms roughness (0 is suitable for this type of calculation), the polarization (1 is suitable) and the grazing incidence angle. Request a data file or request a plot and then click "data file" at the top of the plot. The data column can easily be cut and pasted into the spreadsheet in the columns labeled "WINDOW1, WINDOW2 and MIRROR".

Calculation of the power density deposited and transmitted by the windows and mirror

It is now straightforward to calculate the on-axis power density deposited in the windows and transmitted by them at each energy. It is also simple to sum them to get the total power density deposited on each component and that passed on to the next etc. At the bottom of the columns the total on-axis power density is given together with an angular and a spatial width calculated by the same weighted average as before.

The same procedure applies to the mirror as the windows but note that this calculation deals only with power densities referred to surfaces perpendicular to the beam, the “efficiency factors” characteristic of the optical elements and the inverse-square falloff of power density where applicable. Thus to get the actual power load on the mirror surface you have to supply the geometrical spread factor. Other points to note are as follows.

- In order to make the interesting items at the top and bottom of the spreadsheet be visible and printable on the same page I have hidden many of the rows of the spreadsheet. You will have to unhide them to cut and paste in new data.
- In order to encompass an energy range $0.01\varepsilon_c < \varepsilon < 7\varepsilon_c$ one has to go above the 30 keV limit of the data supplied by CXRO. This can be done by using two tricks: (i) at energies above 30 keV all imaginable beryllium windows are essentially 100% transparent, (ii) at energies far above the critical energy the reflectivity scales inversely as the energy to the fourth power which allows continuation of the mirror data.

How to get and run the spreadsheet

The spreadsheet is in Excel 98 for the Macintosh and can be obtained from the author by accessing the file server “Out box” on the computer called “Malcolm’s Computer” in Appletalk zone Net B128. Log on as Guest, select Out box and the server icon appears on your desk top. Take both the worksheet SR_FUNCS.WSHEET copy and the macrosheet SR_FUNCS.MSHEET copy. To run the calculation launch the macrosheet first. The input data required are in the top left corner of the spreadsheet. The spreadsheet will also run on a Windows-based machine and on earlier versions of Excel. Please contact the author to get such versions.